Gauge Field Theory in the Infrared Regime

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We propose that the low energy behavior of a pure gauge theory can be studied by simply assuming violation of Lorentz invariance which is implemented through a deformation of the canonical Poisson brackets of the theory depending on an infrared scale. The resulting theory is equivalent to a pure gauge theory with a Chern-Simons like term. It is shown that at low energies this theory can be identified with three dimensional QCD where the mass of the fermion is related to the infrared scale.

PACS numbers: 12.38.Lg, 12.38.Aw

The framework of relativistic quantum field theory has been extraordinarily successful in explaining various quantum mechanical phenomena over a wide range of energies. The agreement of the experimental measurements with the results of perturbative calculation for $g-2$ has been remarkable. The predictions of the standard model have also been tested to a very high degree of accuracy and one believes that the discovery of the Higgs particle may be just around the corner.

In spite of all the success, one does not believe that relativistic quantum field theory, in its present form, will be applicable at all energies. There is, of course, the suggestion that at very high energies, string theory will describe all the fundamental interactions of nature in which case we can think of relativistic quantum field theory only as a low energy (compared to the Planck scale) effective field theory. Even if, one does not believe in string theory, it is recognized that in order to be compatible with several gravitational effects and astrophysical observations, relativistic quantum field theory must be modified to incorporate a tiny violation of Lorentz invariance which has been the cornerstone of much of twentieth century physics [1, 2]. At low energies, clearly we do not expect a Lorentz invariant description either. Therefore, there must exist an infrared scale below which a nonrelativistic description should be more meaningful. For example, in an Abelian gauge theory such as QED, at energies much smaller than the electron mass, the theory of electrons interacting with photons can be conveniently described by an effective field theory which leads to a nonrelativistic Schrödinger equation (the photon is always relativistic). Such a procedure, however, cannot be readily carried out in the case of a non-Abelian gauge theory such as QCD which below a certain energy scale (roughly the proton mass) becomes confining. Namely, in this case, in the infrared limit, a nontrivial physical picture arises which necessitates the use of nonperturbative methods since phenomena such as confinement and hadronization are beyond perturbation theory. Unfortunately, nonperturbative techniques are not very well understood as yet.

Precisely in this regime, however, there is a remarkable analogy between molecular physics and effective heavy quark theory following from QCD. Indeed, in molecular physics it is reasonable to assume (as a first approximation) that nuclei are much heavier compared to the electrons and, therefore, are static and as a first approximation, are decoupled from electrons [3]. On the other hand, this assumption (known as the Born-Oppenheimer approximation) cannot be completely right since, as is well known, the electronic and the nuclear variables are coupled through Berry's phase [4]. Following Berry's analysis [3, 4], one finds that at low energies a gauge symmetry emerges as a consequence of global geometrical considerations. This symmetry induces new dynamical effects on the spectrum which has been verified in numerous experiments [5].

Along the same lines one could think that the heavy quark effective field theory [6] (where the heavy quarks are simply assumed to be spectators) should necessarily contain corrections due to the interactions with light quarks as in the case of molecular physics [3]. Although it is technically difficult at the present time to go beyond the Born-Oppenheimer approximation in the context of

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the effective heavy quark field theory, it seems reasonable to explore other possibilities simply by assuming that, in this regime, Lorentz invariance is expected to be violated as a consequence of the infrared scale. Since light quarks are not being integrated out in this approach, the situation may appear to be different from that in molecular physics, nonetheless the approach may represent a step forward in the context of effective heavy quark field theory.

One way to introduce violation of Lorentz invariance into a field theory is by deforming the canonical commutation relations of the theory (so as to have a noncommutative field theory). In two recent papers [7, 8] such a noncommutative gauge field theory violating Lorentz invariance has been studied. The idea is quite simple. One starts with the standard action for a gauge field theory given by

$$S = -\frac{1}{4}\int d^4x \; F^a_{\mu\nu}F^{\mu\nu a}, \tag{1}$$

where the field strength tensor is defined by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \tag{2}$$

with f^{abc} denoting the structure constants of the group. In the context of a noncommutative field approach (inspired by noncommutative geometry), one modifies the conventional canonical (equal-time) Poisson bracket relations on the phase space as

$$\begin{aligned}
\{A^a_\mu(x), A^b_\nu(x')\} &= 0 \to \{A^a_\mu(x), A^b_\nu(x')\} = 0,\\
\{A^a_\mu(x), \Pi^b_\nu(x')\} &= \delta^{ab}\eta_{\mu\nu}\delta^3(x-x') \to \{A^a_\mu(x), \Pi^b_\nu(x')\} = \delta^{ab}\eta_{\mu\nu}\delta^3(x-x'),\\
\{\Pi^a_0(x), \Pi^b_\mu(x')\} &= 0 \to \{\Pi^a_0(x), \Pi^b_\mu(x')\} = 0,\\
\{\Pi^a_i(x), \Pi^b_j(x')\} &= 0 \to \{\Pi^a_i(x), \Pi^b_j(x')\} = \epsilon_{ijk}\theta_k\delta^{ab}\delta^3(x-x'),
\end{aligned} \tag{3}$$

where we have introduced a constant space-like deformation parameter $\theta^\mu = (0, \vec{\theta}\ )$ which has the canonical dimension 1. This modification of the canonical brackets clearly breaks Lorentz invariance and since we would expect a violation of Lorentz invariance only in the infrared sector of the theory, we would like to identify the magnitude of this vector as defining the infrared scale.

It should be noted that the modification of the Poisson bracket relations preserves gauge invariance. Indeed, it can be seen with a little bit of algebra that the theory (1) with the modified Poisson brackets (3) is equivalent to the theory (with the naive canonical Poisson brackets) described by the action

$$S = \int d^4x \left[-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \theta^\mu C_\mu\right], \tag{4}$$

where θ^μ is the constant four-vector introduced earlier (as the deformation parameter) and C_μ is given by

$$C_\mu = \epsilon_{\mu\nu\lambda\rho}\left[A^{\nu a}\partial^\lambda A^{\rho a} + \frac{2g}{3} f^{abc} A^{\nu a} A^{\lambda b} A^{\rho c}\right]. \tag{5}$$

From the form of this action it is clear that while it violates Lorentz invariance (because of the specific form of θ^μ), it is manifestly gauge invariant. However, because of the additional term, the conventional Gauss' law (D_i denotes the covariant derivative)

$$(D_iE_i)^a = (D_iF_{0i})^a = 0, \tag{6}$$

is replaced by in this case by

$$(D_iF_{0i})^a + \epsilon_{ijk}\theta_i F^a_{jk} = (D_iE_i)^a + \theta_i B^a_i = 0. \tag{7}$$

Furthermore, it is clear that if we choose $\theta^\mu = (0,0,0,\theta)$, then the $\theta^\mu C_\mu$ term has the form of the three dimensional Chern-Simons term [9] (although the fields live in four dimensions). It should be stressed that the approach discussed above is consistent with that proposed in [10], in which Lorentz invariance is essentially broken through the addition of the Chern-Simons term to a Maxwell or Yang-Mills theory coupled to an external four vector field.

The purpose of this short note is to propose the model defined by the action (4) as an example of QCD at low energy (low compared to the scale θ) within an alternative approach. We note that one can try to derive (4) as an effective theory by introducing fermion fields such that when they are integrated out one ends with a Lorentz and CPT violating Chern-Simons term. This has been achieved in [11]-[13] by including a coupling to an axial-vector term which violates Lorentz invariance in an extended Dirac Lagrangian. In fact, originally, the interest in theories containing a Chern-Simons term was triggered by the observation that such a term is induced in the effective action of the gauge field through fluctuations of massive fermion fields. This is the celebrated parity anomaly [9, 14]. Another possibility was discussed in [15] by studying a theory with $SU(2)_L$ gauge fields and a finite chemical potential term which can be interpreted as the coupling of an external $U(1)$ gauge field to a $U(1)$ current. Then, as a consequence of the $U(1)_L$ anomaly of such a current, a Chern-Simons term is generated. In the following, we will take an approach related to this latter case.

Let as add to the Lagrangian density of QCD a gauge invariant chiral fermion term of the form

$$\mathcal{L}_{\mathrm{f}} = i\bar{\psi}_L \not{D} \psi_L \,,$$

with ψ_L a left handed Weyl spinor, so that the complete Lagrangian density reads

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + i\bar{\psi}_L \not{D} \psi_L . \tag{8}$$

Integrating out the chiral fermions would yield an effective action of the form

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \log\det\left[i\not{D}\right] . \tag{9}$$

Now, for Weyl fermions the Dirac operator does not have a well defined eigenvalue problem since it maps negative chirality spinors into positive chirality ones. This implies that the definition of the fermion determinant in (9) is, in general, problematic. One way out is to modify the gauge covariant Dirac operator by adding free right-handed fermions into the theory so that one ends up with

$$\not{D} = (\not{\partial} - ig\not{A})\, P_- \to D[A] = \not{\partial} - ig\not{A} P_- , \tag{10}$$

where P_- denotes the projection operator onto the left-handed spinor space

$$P_- = \frac{1-\gamma_5}{2} . \tag{11}$$

The operator $D[A]$ does define an eigenvalue problem since it acts on Dirac fermions *i.e.*

$$iD[A]\varphi_n = \lambda_n \varphi_n , \tag{12}$$

so that the fermion determinant can be computed as the product of eigenvalues in the usual way. However, since the Dirac operator is unbounded, the product of eigenvalues needs to be regularized. Furthermore, since the right-handed fermions are decoupled from the gauge field, the modified operator (10) is not gauge covariant as the original one was. Hence, there is no way to obtain a gauge invariant answer for the regularized fermion determinant. It is through this mechanism that an anomalous term is generated (see for example [16] and references therein).

Let us note that (we use the convention $\left(\gamma^0\right)^\dagger = \gamma^0, \left(\gamma^i\right)^\dagger = -\gamma^i, \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = (\gamma_5)^\dagger$)

$$\left(\gamma_5\gamma^3\right)^\dagger = \gamma_5\gamma^3, \quad \left(\gamma_5\gamma^3\right)^2 = 1, \quad \det\left(\gamma_5\gamma^3\right) = 1, \tag{13}$$

so that we can write

$$\det\left(iD[A]\right) = \det\left(i\gamma_5\gamma^3 D[A]\right) . \tag{14}$$

Returning to our discussion of the infrared sector of QCD let us choose, as suggested earlier, $\theta^\mu = (0,0,0,\theta)$ in (4). Moreover, we partially fix the gauge degrees of freedom through the condition $A_3 = 0$. With this, the eigenfunctions (12) can be expanded in an interval $0 \leq x^3 \leq 1/\theta$ with anti-periodic boundary condition[20] as

$$\varphi_n(x^0,x^1,x^2,x^3) = \sum_{m=-\infty}^{\infty} e^{i(2m+1)\pi\theta x^3} \tilde{\varphi}_{n,m}(x^0,x^1,x^2). \tag{15}$$

We note that, for a fixed value of m, the operator $i\gamma_5\gamma^3 D[A]$, in the space of functions $\tilde{\varphi}_{n,m}$, has the form

$$i\gamma_5\gamma^3 D[A] = \left(i\gamma_5\gamma^3\gamma^\alpha(\partial_\alpha - igA_\alpha P_-) + M(m)\gamma_5\right) , \tag{16}$$

where $\alpha = 0,1,2$ and we have identified $M(m) = (2m+1)\pi\theta$. For simplicity and clarity, let us choose the Weyl basis (chiral basis) for the representation of the gamma functions,

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}, \tag{17}$$

where $\sigma_\mu = (I, \sigma_i), \bar{\sigma}_\mu = (I, -\sigma_i)$ with I denoting the two dimensional unit matrix. In this representation, Eq. (16) has the explicit form

$$\begin{aligned} & i\gamma_5\gamma^3 D[A] \\ = & \begin{pmatrix} -i\sigma_3\bar{\sigma}_\alpha\left(\partial_\alpha - igA_\alpha\right) - M(m) & 0 \\ 0 & -i\sigma_3\sigma_\alpha\partial_\alpha + M(m) \end{pmatrix} . \end{aligned} \tag{18}$$

This shows that the right handed fermions decouple and lead only to a constant multiplicative factor in the evaluation of the determinant. Furthermore, identifying the 2×2 Dirac matrices of three dimensional space-time as

$$\gamma^\alpha_{(3)} = -\sigma_3\bar{\sigma}_\alpha, \quad \alpha = 0,1,2, \tag{19}$$

we obtain, for any fixed m,

$$\begin{aligned} \det\left(iD[A]\right) &= \det\left(i\gamma_5\gamma^3 D[A]\right) \\ &= \mathcal{N}\det\left(i\not{D}^{(3)} - M(m)\right) . \end{aligned} \tag{20}$$

Thus, for energies much lower than the infrared scale, $E \ll \theta$, only the lowest mode ($m=0$) contributes and one finds that the four and the three dimensional determinants coincide up to a multiplicative factor arising from the decoupled right handed sector,

$$\det\left(i\not{D}\right)^{(4)} = \mathcal{N}\det\left(i\not{D} - \pi\theta\right)^{(3)} , \tag{21}$$

which is an important result for our work. Furthermore, the three dimensional determinant for a massive fermion (with mass $\pi\theta$) is known to generate a Chern-Simons term which is how the action (4) can be obtained as an effective action.

Indeed, at low energies, we can generalize this equivalence by scaling

$$\begin{aligned} &\psi_L \to \sqrt{\theta}\psi , \\ &g^{(4)} \to \left(\sqrt{\theta}\right)^{-1} g^{(3)} , \end{aligned} \tag{22}$$

One can then describe the pure gauge theory with a term violating Lorentz invariance as in (4), in terms of relativistic three dimensional QCD with fermions having a mass related to the infrared scale, namely

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \bar{\psi}\left(i\not{D}^{(3)} - \pi\theta \right)\psi. \qquad (23)$$

In summary, in this paper we have argued that a four dimensional gauge theory with a Chern-Simons term could be considered as bosonized QCD at low energy. Although in four dimensions this theory violates Lorentz invariance, nonetheless one can map the gauge theory with a Chern-Simons term to relativistic QCD in three dimensions. Thus, the point of view advocated in this paper would suggest a new route for an old problem in the nonperturbative sector of the gauge theory. We expect to compare the abundant results in three dimensional QCD [18] with heavy quark theory [19] in a future publication.

This work was supported in part by US DOE Grant number DE-FG 02-91ER40685, FONDECYT 1050114, MECESUP-0108, PIP-CONICET 6160, and CICBA and UNLP grants.

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